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LETTER TO THE EDITOR

# Critical behaviour of anisotropic spiral self avoiding walks

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**Abstract.** Here we consider three-choice-spiral and two-choice-spiral self avoiding walks (SAWS) on a square lattice, for which the (anisotropic) constraints are such that for any $n$th step in the $x$ and $-x$ direction, the $(n+1)$th step is forbidden to be in the clockwise (or anticlockwise) direction, while for the $n$th step in the $y$ and $-y$ direction, there may or may not be any special constraints for the next step. In the case of three-choice-spiral SAWS, there is no such constraint (other than self avoiding) while, in the case of two-choice-spiral SAWS, with the $n$th step in the $y$ and $-y$ directions, the $(n+1)$th step is forbidden to follow the same direction. From exact enumeration results we find a new universality class having $\gamma=1.58\pm0.05$ and $\nu=0.842\pm0.014$ for such SAWS.

Recently considerable interest has been shown in studying the effect of different constraints (microscopic or macroscopic) on self avoiding walk (SAW) statistics, specifically in two dimensions. For example, Grassberger (1982) showed that two-choice SAWS, in which no two successive steps are allowed in the same lattice direction (microscopic constraint), belong to the same universality class as that of ordinary SAWS. Directed self avoiding walks (DSAWS) (Fisher and Sykes 1959, Chakrabarti and Manna 1983) are forbidden to have any steps along a specified lattice direction (macroscopic constraint). Field-theoretic and exact studies (Cardy 1983, Redner and Majid 1983) show that the critical behaviour of DSAWS is mean-field-like and is anisotropic. In spiral SAWS (Privman 1983), the constraint is such that every step forbids its next step to be in the clockwise (or anticlockwise) direction, so that the walk spirals about a direction perpendicular to the plane of the walk. Such a macroscopic constraint also leads to a different universality class (Privman 1983).

Here we have studied some anisotropic hybridisation of such constraints, which lead to a new universality class for the constraint SAWS. Specifically we have considered SAWS on a square lattice where any $n$th step in the $x$ (and $-x$) direction forbids the next $(n+1)$th step to be in the clockwise (or anticlockwise) direction (spiral constraint for the $(n+1)$th step when the $n$th step is in the $x$ and $-x$ directions) while for steps in the $y$ (and $-y$) directions there may or may not be special constraints for the next step. In the first case, which we shall call three-choice-spiral SAWS, there is no constraint (other than the self avoiding restriction) for the $(n+1)$th step, when the $n$th step is in the $y$ (and $-y$) directions (see figure 1); while for the second case, which we shall call two-choice-spiral SAWS, the $(n+1)$th step is forbidden to be along the $y$ direction (two-choice constraint) if the preceding $n$th step is in the $y$ (and $-y$) direction (see figure 1). From the exact enumeration results, we find a new universality class for SAWS with such constraints.

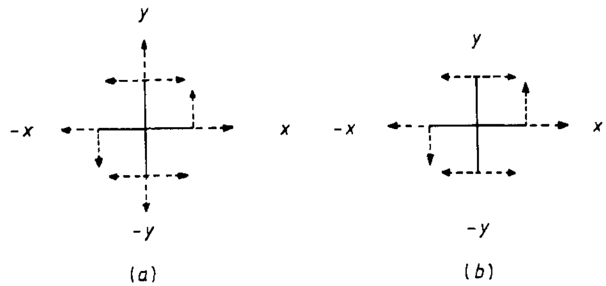


**Figure 1.** Choices for the $(n+1)$th step (shown by broken arrows) for various directions of the $n$th step (shown by full lines). ($a$) for three-choice-spiral SAWs ($b$) for two-choice-spiral SAWs.

The total number of independent configurations $G_N$ of such special spiral SAWs of $N$ steps are given in tables 1 and 2 (for three-choice and two-choice-spiral SAWs respectively) from exact enumeration. In order to see the possible anisotropy in the shape of an average configuration of such spiral SAWs, we have calculated the average of the projection of the end-to-end distance squares $\langle R_N^2\rangle$, for the $N$-stepped walks, along the $x$ axis, $y$ axis, $y=x$ line and $y=-x$ line ($\alpha=1$, 2, 3 and 4 directions respectively). These are also given in tables 1 and 2. Both $G_N$ and $\langle R_N^2\rangle$ (angular bracket denoting configurational average) are assumed, in the $N\to\infty$ limit, to have the scaling forms (but see later)

$$G_N \sim \mu^N N^{\gamma-1} \tag{1}$$

$$\langle R_N^2\rangle \sim N^{2\nu} \qquad (\alpha=1,\ldots,4). \tag{2}$$

**Table 1.** Simulation results for $G_N$ and $\langle R_N^2\rangle_\alpha$, $\alpha=1$, 2, 3 and 4 for $N$ up to 21 for three-choice-spiral SAWs.

| $N$ | $G_N$ | $\langle R_N^2\rangle_{x \text{ axis and } y \text{ axis}}$ | $\langle R_N^2\rangle_{y=x}$ | $\langle R_N^2\rangle_{y=-x}$ |
|---|---|---|---|---|
| 1 | 4 | 0.500 00 | 0.707 11 | 0.707 11 |
| 2 | 10 | 1.400 00 | 1.600 00 | 1.200 00 |
| 3 | 24 | 2.500 00 | 3.166 66 | 1.833 33 |
| 4 | 54 | 3.925 92 | 5.333 33 | 2.518 52 |
| 5 | 124 | 5.403 22 | 7.725 80 | 3.080 64 |
| 6 | 272 | 7.250 00 | 10.764 70 | 3.735 29 |
| 7 | 608 | 9.065 78 | 13.868 42 | 4.263 15 |
| 8 | 1 314 | 11.287 67 | 17.680 36 | 4.894 97 |
| 9 | 2 884 | 13.426 49 | 21.448 68 | 5.404 29 |
| 10 | 6 178 | 15.978 31 | 25.944 96 | 6.011 65 |
| 11 | 13 388 | 18.424 11 | 30.339 25 | 6.508 96 |
| 12 | 28 486 | 21.284 42 | 35.472 30 | 7.096 53 |
| 13 | 61 168 | 24.019 35 | 40.455 00 | 7.583 70 |
| 14 | 129 446 | 27.173 13 | 46.190 87 | 8.155 40 |
| 15 | 276 020 | 30.184 02 | 51.733 96 | 8.634 07 |
| 16 | 581 572 | 33.617 69 | 58.043 16 | 9.192 23 |
| 17 | 123 320 4 | 36.894 14 | 64.124 53 | 9.663 75 |
| 18 | 2 588 906 | 40.596 82 | 70.983 37 | 10.210 27 |
| 19 | 5 464 816 | 44.129 53 | 77.583 56 | 10.675 49 |
| 20 | 11 437 088 | 48.092 10 | 84.972 29 | 11.211 91 |
| 21 | 24 050 760 | 51.873 34 | 92.075 12 | 11.671 56 |

**Table 2.** Simulation results for $G_N$ and $\langle R_N^2 \rangle_\alpha$, $\alpha = 1$, 2, 3 and 4 for $N$ up to 28 for two-choice-spiral SAWs.

| $N$ | $G_N$ | $\langle R_N^2 \rangle_{x\text{-axis}}$ | $\langle R_N^2 \rangle_{y\text{-axis}}$ | $\langle R_N^2 \rangle_{y=x}$ | $\langle R_N^2 \rangle_{y=-x}$ |
|---|---|---|---|---|---|
| 1 | 4 | 0.500 00 | 0.500 00 | 0.707 11 | 0.707 11 |
| 2 | 8 | 1.750 00 | 0.750 00 | 1.500 00 | 1.000 00 |
| 3 | 16 | 3.375 00 | 1.125 00 | 3.000 00 | 1.500 00 |
| 4 | 28 | 5.928 57 | 1.642 85 | 5.285 71 | 2.285 71 |
| 5 | 52 | 8.576 92 | 2.115 38 | 7.730 76 | 2.961 53 |
| 6 | 90 | 12.200 00 | 2.777 77 | 11.066 66 | 3.911 11 |
| 7 | 160 | 15.900 00 | 3.400 00 | 14.500 00 | 4.800 00 |
| 8 | 276 | 20.413 04 | 4.181 15 | 18.695 65 | 5.898 55 |
| 9 | 484 | 24.900 82 | 4.909 09 | 22.880 16 | 6.929 75 |
| 10 | 826 | 30.368 03 | 5.825 66 | 27.985 47 | 8.208 23 |
| 11 | 1 434 | 35.601 11 | 6.659 69 | 32.884 93 | 9.375 87 |
| 12 | 2 438 | 41.842 49 | 7.680 06 | 38.731 74 | 10.790 81 |
| 13 | 4 194 | 47.845 49 | 8.626 13 | 44 366 47 | 12.105 15 |
| 14 | 7 104 | 54.832 20 | 9.748 87 | 50.927 36 | 13.653 71 |
| 15 | 12 150 | 61.518 18 | 10.793 08 | 57.215 06 | 15.096 21 |
| 16 | 20 506 | 69.243 05 | 12.018 82 | 64.481 42 | 16.780 45 |
| 17 | 34 898 | 76.605 53 | 13.161 04 | 71.415 92 | 18.350 65 |
| 18 | 58 740 | 85.018 35 | 14.482 83 | 79.339 87 | 20.161 32 |
| 19 | 99 568 | 93.031 17 | 15.719 10 | 86.895 87 | 21.854 41 |
| 20 | 167 186 | 102.115 91 | 17.135 69 | 95.462 29 | 23.789 31 |
| 21 | 282 468 | 110.753 57 | 18.462 21 | 103.615 31 | 25.600 47 |
| 22 | 473 318 | 120.485 32 | 19.970 35 | 112.800 24 | 27.655 43 |
| 23 | 797 462 | 129.732 52 | 21.385 07 | 121.535 79 | 29.581 80 |
| 24 | 1 333 866 | 140.091 39 | 22.982 26 | 131.320 18 | 31.753 47 |
| 25 | 2 241 980 | 149.930 85 | 24.482 65 | 140.621 62 | 33.791 82 |
| 26 | 3 744 048 | 160.900 06 | 26.166 79 | 150.989 50 | 36.077 35 |
| 27 | 6 279 996 | 171.317 00 | 27.750 74 | 160.842 71 | 38.225 04 |
| 28 | 10 472 560 | 182.880 79 | 29.519 78 | 171.789 61 | 40.621 61 |

**Table 3.** Extrapolated values of $\mu$, $\gamma$ and $\nu$ (equations (1) and (2)) for three and two-choice-spiral SAWs.

| | | | $\nu_1$ | $\nu_2$ | $\nu_3$ | $\nu_4$ |
|---|---|---|---|---|---|---|
| Three-choice-spiral SAW | 2.04 | 1.613 | $0.828 \pm 0.001$ | $0.828 \pm 0.001$ | $0.847 \pm 0.001$ | $0.428 \pm 0.001$ |
| Two-choice-spiral SAW | 1.63 | 1.535 | $0.852 \pm 0.002$ | $0.852 \pm 0.001$ | $0.854 \pm 0.001$ | $0.834 \pm 0.001$ |

The $\mu$ and $\gamma$ values are obtained following the extrapolation procedure outlined by Martin (1967) and to find $\nu$, we define (Grassberger 1982) $\nu_N = (N/2)[(\langle R_{N+1}^2 \rangle / \langle R_N^2 \rangle) - 1]$ for each direction $\alpha$ and take the extrapolated $\nu_N$ values in the limit $N^{-1} \to 0$ (see figures 2 and 3). The values of $\mu$, $\gamma$ and $\nu_1$, $\nu_2$, $\nu_3$ and $\nu_4$ are given in table 1.

The values of $\gamma$ and $\nu$, as given in table 3, clearly indicate a new universality class for such SAWs, different from those of ordinary SAWs ($\gamma = 1.34$, $\nu = 0.75$) or of spiral SAWs ($\gamma = 5.2 \pm 1.3$, $\nu = 0.62 \pm 0.06$).

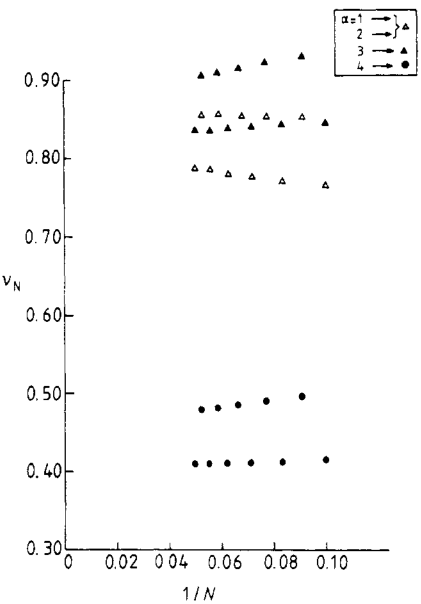


**Figure 2.** Plot of $\nu_N$ against $1/N$ for three-choice-spiral SAWS.

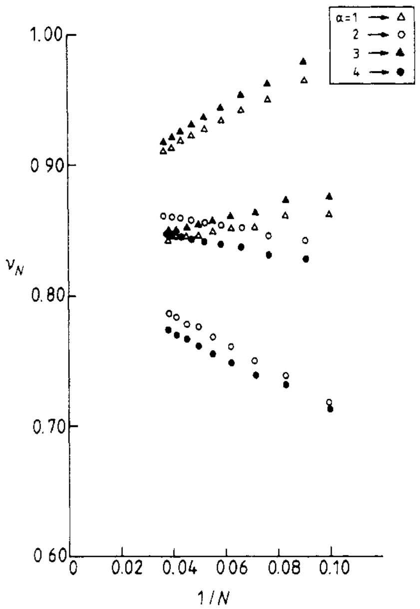


**Figure 3.** Plot of $\nu_N$ against $1/N$ for two-choice-spiral SAWS.

The critical behaviour of the statistics of two-choice-spiral SAWS is thus seen to be isotropic ($\nu = 0.842 \pm 0.014$) within limits of experimental accuracy, while that of three-choice-spiral SAWS is anisotropic with the same value of $\nu$ in the $y = x$ direction and with $\nu = 0.428 \pm 0.001$ in the perpendicular direction. However, in view of the apparent 'microscopic' anisotropy of the definition of such SAWS (both three and two-choice-spiral SAWS) this is rather surprising. It may be noted that this is not peculiar to two-choice-spiral SAWS alone: one can also define three-choice-two-choice SAWS where for the $n$th step in the $x$ and $-x$ directions, the SAWS cannot follow the same direction for the $(n+1)$th step, while for steps in the $y$ and $-y$ directions there are no restrictions (other than self avoiding) for the next step. Exact enumeration results show the critical behaviour of such walks also to be isotropic, with the same exponent value as those of ordinary SAWS (Manna 1984).

After completion of this work we came across the work of Blöte and Hilhorst (1984), and of Guttmann and Wormald (1984), which shows that the statistics of spiral SAWS is exactly solvable and the assumed scaling forms (1) and (2), for $G_N$ and $\langle R_N^2 \rangle$ respectively, are not strictly valid for spiral SAWS. This is particularly so for $G_N$ while for $\langle R_N^2 \rangle$, the scaling form (2) is valid with a logarithmic correction of $\nu = 1/2$ (Redner and de Arcangelis 1984, Blöte and Hilhorst 1984). We therefore fitted the values of $\langle R_N^2 \rangle_\alpha / \log(N)$ to a form $N^{2\nu'}$ for all $\alpha$'s. For three-choice-spiral SAWS, $\nu_1'$, $\nu_2'$, $\nu_3'$ are $0.723 \pm 0.009$ and $\nu_4$ is $0.311 \pm 0.001$. For two-choice-spiral SAWS for all directions $\nu'$ is $0.735 \pm 0.015$.

As already mentioned (see table 3), without the $\log(N)$ term $\langle R_N^2 \rangle_\alpha$ fits the form $N^{2\nu}$, with $\nu_1$, $\nu_2$, $\nu_3$ equal to $0.842 \pm 0.014$ and $\nu_4 = 0.428 \pm 0.001$ for three-choice-spiral SAWS and $\nu = 0.842 \pm 0.014$ for all directions for two-choice-spiral SAWS. Since the errors in the fitting values of $\nu$ and $\nu'$ are of the same order, we cannot distinguish here between the two scaling forms. However, in either of the two cases, the observed $\nu'$ or $\nu$ values (with and without the log correction) do not fall into any of the known universality classes.

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